[redacted]

NASA TT F-8256

TO THE THEORY OF ELECTROMAGNETIC WAVE PROPAGATION IN A PIECEWISE-UNIFORM PLANE WAVEGUIDE

by YU. K. Kalinin.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON JULY 1962

X63 10460

DRAFT TRANSLATION NASA TT F-8256

JUL 9 1962

TO THE THEORY OF ELECTROMAGNETIC WAVE PROPAGATION IN A PIECEWISE-UNIFORM PLANE WAVEGUIDE

(K teorii rasprostraneniya elektromagnitnykh voln v kusochno-odnorodnom volnovode)

Geomagnetizm i Aeronomiya
Tom II, No.1, 80—85,
Izd-vo A.N.SSSR, 1962.

by Yu. K. Kalinin

ABSTRACT

22856 10468

Examined is the field of a point emitter in a piecewise-uniform waveguide with the help of approximate impedance-type boundary conditions. A formula is obtained for a layout consisting of two extended sections. An estimate is made of the effect of geometrical inhomogeneities.

Author

COVER TO COVER TRANSLATION

Approximate Boundary Conditions

As is well known [1], the classical way for an exact solution of a boundary problem requires a simultaneous consideration of fields on both sides of the boundary of the partition, with a subsequent joining of the fields at that boundary. However, this is not the only way of doing it. It is possible to utilize such approximate impedance-type boundary conditions [2—4], when the exact solution resides in the consideration of the field within the bounded region of space. The Green equation is in that case the most appropriate device.

This Green equation is an integral equation written as follows :

.../..

$$\psi = \psi_0 + \frac{1}{4\pi} \int_S \left[\frac{\partial \psi}{\partial n} \psi_0 - \psi \frac{\partial \psi_0}{\partial n} \right] dS, \tag{1}$$

where ψ_0 is the Green function ; S — the surface of the division n — the normal to its boundary ; ψ is the scalar potential or one of the components of the Hertz vector. Considered are below only axially-symmetric problems in which one component of the Herz vector is sufficient. Together with that, ψ may be in certain cases one of the field's components. Let us write the approximate boundary condition in the form :

$$\frac{\partial \psi}{\partial n} = \alpha \psi, \quad \alpha = -i \frac{k_i^2}{k_2^2} \sqrt{k_2^2 - k_i^2 \cos \vartheta}, \tag{2}$$

where k_i and k_2 - are the wave numbers at different sides of the division's boundary ; ϑ is the angle of slide of the incident wave. The boundary condition (2) is exact for a plane wave and approximate for the arbitrary-type wave.

Equation (1) is valid only for waves that vary in the neighborhood of the source as **1/r**. Thus, if ψ represents the field, it may only be equal to the wave part of the total field. And if the solution is sought for in the form of Fourier spectrum, the field's wave part may only be found with the aid of equations (1) and (2). But it is well known that a precise description of the field is possible only with the help of the potential and that in case of vectorial fields the condition (2) determines only the normal component of the field to the division's boundary. Consequently, the approach used here is possible, if the potential, fully determining the field, has one normal component, i.e. the vector-potential must have the form $\boldsymbol{\psi} = \psi_n \mathbf{i}_n + 0 \mathbf{i}_\tau$, where $\mathbf{i}_n$ and $\mathbf{i}_\tau$ are the normal and the tangential crosscuts. The field **U** is linked with the vector-potential by the formula

$$\mathbf{U} = -k^2 \boldsymbol{\psi} + \mathbf{grad\,div}\, \boldsymbol{\psi}. \tag{3}$$

Obviously, the first component represents in this formula the wave part of the field. Therefore, formulas (1) and (2) allow the finding of the exact solution in the bounded region either for Ψ or (which is the same) for the wave part of the field.

As an example of the obtention of such a solution with the help of formulas (1) and (2), let us consider the well known problem of the dipole field in a plane waveguide. Formulas (1) and (2) permit the easy passing from the solution in case of ideally-conducting walls ($\alpha = 0$) to the case when one of the walls is endowed with a finite conductivity ($\alpha \neq 0$). For this, it is necessary to seek a solution in the form of two-dimensional Fourier integrals. But it then is easy to pass from the equation (1) to the equation for the spectra $S[\Psi]$ and the $S[\Psi_0]$ – functions, two-dimensionally conjugated by Fourier in regard to functions Ψ and Ψ_0. For the spectra, we shall obtain the following equation [5]:

$$S[\psi] = S[\psi_0] + S[\psi_0]\, S\left[\psi \frac{1}{\psi}\ \frac{\partial \psi}{\partial n}\right]. \qquad (4)$$

For Ψ_0 we shall select a field in a waveguide in whose walls $\alpha = 0$. Let us assume that $\alpha = \alpha_1$ on the upper wall of the waveguide, i.e. that the second component of the right-hand part of formula (4) must be computed on the upper wall of the waveguide. If the source is disposed at the origin of cylindrical coordinates on the lower wall of the waveguide, $S[\Psi_0]$ is expressed by the formula

$$S[\psi_0] = \frac{\operatorname{ch}[(H-z)\,b]}{b \operatorname{sh} Hb}, \qquad (4a)$$

where H is the height of the waveguide; b is $ik_0 \cos\theta$; θ is the angle of incidence. It is then appropriate to seek $S[\Psi]$ in the form

$$S[\psi] = \frac{1}{b}\ \frac{\exp[b(H-z)] - B\exp[-b(H-z)]}{\exp[b(H-z)] + B\exp[-b(H-z)]}, \qquad (5)$$

where B is a factor subject to determination.

Such spectrum selection is quite natural, for it assures the conversion to zero of the normal derivative ψ at the lower boundary and constitutes a natural generalization of formula (4). The substitution of formulas (2) and (5) into the equation (4), which takes the form

$$S[\psi]_{z=0} = S[\psi_0]_{z=0} + \alpha_1 S[\psi_0]_{z=H} \cdot S[\psi]_{z=H},$$

allows to determine the coefficient B. As this was to be expected B is equal to the Fresnel reflection coefficient from the upper boundary, taken with the minus sign. One may analogously pass to a waveguide at whose lower boundary* $\alpha \neq 0$.

Considered below is a waveguide which is non-homogenous in the horizontal direction. In the particular case of a piecewise-uniform waveguide, the approximate solution far off the boundaries of uniform portions, is searched for with the help of the equations (1) and (2). The Green equation is resolved by the iteration method [5]. It should be noted, that it would have been more sequential to utilize the boundary conditions in the form (1), so that the transition to the approximate representation $\alpha \approx -i(k_1/k_2)$ be materialized in the obtained iteration solution. Such method would allow, at least in principle, to estimate the error resulting from the approximate representation of α.

Piecewise-Uniform Waveguide

Let be two plane-parallel boundaries with a distance h between them. The reflection coefficient from the upper plane is equal to $R(\nu)$, where ν – is the angle of slide cosine. Boundary conditions (2) are fulfilled on the lower boundary, so that

$$\alpha = \begin{cases} \alpha_1 & 0 < r < r_H \\ \alpha_2 & r_H < r < D, \end{cases} \qquad (6)$$

where r_H – is the nonuniformity limit and D is the distance to the point of observation.

In order to find ψ, let us utilize the Green equation (1). The total integration surface S consists of S_1 and S_2 (respectively upper and lower boundaries); S_2 consists in its turn of S_{21} and S_{22} (first and second uniform sections). For ψ_0 it is appropriate to select a uniform solution in the second section ψ_2. Equation (1) then takes the form :

$$\psi = \psi_2 + \frac{1}{4\pi} \int\limits_{S_1+S_2} \left[\frac{\partial \psi}{\partial n} \psi_2 - \psi \frac{\partial \psi_2}{\partial n} \right] dS. \tag{7}$$

If one assumes, that within the limits of essential integration region, quantities ψ and ψ_2 satisfy approximately the boundary conditions (2), the integral along S_1 is zero. Then, utilizing the link between ψ_1 and ψ_2, valid with a precision to the condition (2),

$$\psi_1 = \psi_2 + \frac{1}{4\pi} \int\limits_{S_2} \left[\frac{\partial \psi_1}{\partial n} \psi_2 - \psi_1 \frac{\partial \psi_2}{\partial n} \right] dS, \tag{8}$$

one may obtain for ψ an approximate expression in the form of the first two terms of the iteration series :

$$\psi \approx \psi_1 - \frac{1}{4\pi} \int\limits_{S_{22}} \left[\frac{\partial \psi_1}{\partial n} \psi_2 - \psi_1 \frac{\partial \omega_2}{\partial n} \right] dS. \tag{9}$$

This expression is also obtained with applying condition (2) to the total values ψ and ψ_0 on the upper wall. For that it is necessary to substract formula (8) from formula (7). Integration must also be effect along $S_1 + S_2$ in formula (8). We shall obtain as a result, an equation for ψ, in which no neglect of any sort has been made as yet :

$$\begin{aligned} \psi = \psi_1 &- \frac{1}{4\pi} \int\limits_{S_{22}} \left[\frac{\partial \psi_2}{\partial n} \psi_2 - \psi_1 \frac{\partial \psi_2}{\partial n} \right] dS + \\ &+ \frac{1}{4\pi} \int\limits_{S_{21}} \left[\frac{\partial (\psi - \psi_1)}{\partial n} \psi_2 - (\psi - \psi_1) \frac{\partial \psi_2}{\partial n} \right] dS + \\ &+ \frac{1}{4\pi} \int\limits_{S_1} \left[\frac{\partial (\psi - \psi_1)}{\partial n} \psi_2 - (\psi - \psi_1) \frac{\partial \psi_2}{\partial n} \right] dS. \end{aligned}$$

If we now apply the iteration method in the right-hand part, we shall obtain formula (9), since the integrals along S_{21} and S_1 will convert to zero. Thus, the fact that the total quantities ψ_2 and ψ_1 satisfy approximately the condition (2) near the upper boundary, will only take effect in terms proportional to $(\alpha_1 - \alpha_2)^2$.

Let us now transform formula (9), using the following expression for ψ_j :

$$\psi_j = \frac{ik_0}{4} \int_{\Gamma} \frac{H_0^{(1)}(k_0^2 \sqrt{1-\nu^2})(1+R_j)(1+F)}{1-R_j F} d\nu, \qquad (10)$$

where R_j is the Fresnel reflection factor from the lower boundary, which in case $|k_j/k_0| \gg 1$ (k_j - being the wave number un the space $z<0$ may considered equal to $R_j \approx 1 - (2k_0/k_j\nu)$. The value F is determined from the formula

$$F = \tilde{R}(\nu)\exp(2ik_0\nu h),$$

where $\tilde{R}(\nu)$ — is the reflection factor from the upper boundary. In formula (9), ψ_1 is taken from the argument r, and ψ_2 — from the argument

$$\rho = \sqrt{(D-r)^2 + Dr\sin^2\frac{\beta}{2}},$$

where r and β are the polar coordinates in the plane $z = 0$; $dS = r dr d\beta$. In order to integrate along S_{22} one may approximately assume $|\beta| \ll 1$, $r_н < r < D$.

Function $H_0^{(1)}$ in formula (10) shall be substituted by its asymptotic expression [7]:

$$H_0^{(1)}(x) \approx \sqrt{\frac{2}{\pi x}} e^{i\left(x-\frac{\pi}{4}\right)}. \qquad (11)$$

X63-10468

Taking advantage of the exact boundary conditions (2), we shall represent formula (9) in the form:

$$\psi = \psi_1 - \frac{ik}{4\pi}\int\limits_{S_{22}} dS \int\limits_{\Gamma_1}\int\limits_{\Gamma_2} \frac{H_0^{(1)}(k_0 r\sqrt{1-\nu^2})(1+R_1)(1+F)}{1-R_1F} \times$$

$$\times \frac{H_0^{(1)}(k_0\rho\sqrt{1-\mu^2})(1+R_2)(1+F)}{1-R_2F}\left[\frac{ik_0^2\sqrt{k_{21}^2/k_0^2-\nu^2}}{k_{21}^2} - \frac{ik_0^2\sqrt{k_{22}^2/k_0^2-\nu^2}}{k_{22}^2}\right] d\mu\, d\nu.$$

Let us substitute the sub-integral functions by their asymptotic expressions according to forumla (11), and let us integrate along β using the stationary phase method. Then

$$\psi = \psi_1 - \frac{ik_0^2}{4\pi}\int\limits_{r_0}^{D} dr\, \frac{\pi i}{2k_0}\sqrt{\frac{2\pi i}{4D}} \times$$

$$\times \int\limits_{\Gamma_1}\int\limits_{\Gamma_2} \frac{\exp\left[ik\left(r\sqrt{1-\nu^2}+(D-r)\sqrt{1-\mu^2}\right)\right]}{\sqrt[4]{1-\nu^2}\sqrt{1-\mu^2}\,[1-R_1F]\,[1-R_2F]}\left[\frac{\sqrt{\frac{k_{21}^2}{k_0^2}-\nu^2}}{k_{21}^2} - \frac{\sqrt{\frac{k_{22}^2}{k_0^2}-\mu^2}}{k_{22}^2}\right] d\mu\, d\nu.$$

Upon integration along **r** and computation of the dual series of deductions, the double integral integral will convert into the sum of two terms, one of which will be reduced by Ψ_1, and we shall have for Ψ:

$$\psi = \frac{1}{\sqrt{r_1+r_2}}\sqrt{\frac{2}{\pi i}}\left(\frac{k_0}{k_{21}} - \frac{k_0}{k_{22}}\right) \times$$

$$\times \sum_{n,\,m}' \frac{\exp\left[ik_0\left(r_1\sqrt{1-\nu_n^2}+r_2\sqrt{1-\mu_m^2}\right)\right](1-\nu_n^2)^{1/4}(1-\mu_m)^{1/4}}{(R_1F)'_{\nu=\nu_n}(R_2F)'_{\mu=\mu_m}(\nu_n-\mu_m)}, \qquad (12)$$

where r_1 and r_2 are the lengths of the first and second portions; ν_n and μ_m are the radicals of the equations $R_1F = 1$ and $R_2F = 1$. The structure of formula (12) is analogus to that of the correspon- ding formula in case of terrestrial wave propagation over a spheri- cal piecewise-uniform layout [5]. Formula (12) satisfies three threshold transitions: at $r_{1,2} \to 0$ it passes to the formula for $\Psi_{1,2}$, at $k_{12} \to k_{22}$ is passes to the formula for a uniform Ψ_i from the argument of $r_1 + r_2$.

To demonstrate these transitions it is necessary to pass again from a dual sum to a double integral. Subsequently, if for example $r_2 \to 0$, it is necessary to represent the denominator in the form $1 - R_2F = 1 - R_1F + (R_1 - R_2)F$. Then, deforming the integration contour encompassing the radicals of the equation $1 - R_2F = 0$ so that it embrace the only complementary pole $\nu = \mu$, one may obtain that the integral along μ be :

$$\frac{1}{\nu}\left(\frac{k_1}{k_{21}} - \frac{k_1}{k_{22}}\right)\frac{2F^{-1}}{R_1 - R_2}.$$

Taking into account now, that $R_1 \approx 1 - (2k_1/\nu k_{21})$, $R_2 \approx 1 - (2k_1/\nu k_{22})$, we shall obtain that this quantity is equal to the unity. In the case when $k_{21} \to k_{22}$, only diagonal terms shall remain in the dual sum. After similar transformations with the integral along , the result of integration will give :

$$\frac{2}{\nu}\left[\frac{\partial (R_1 - R_2)}{\partial\left(\frac{k_1}{k_{21}} - \frac{k_1}{k_{22}}\right)}\right]^{-1}$$

thus also the unity. This is precisely the manner in which it was deminstrated at formula (12) deduction, that upon inegration along r the upper boundary gives a component, which will be reduced from Ψ_1. It should be noted that at formula (12) deduction it was assumed that the only particular points in the in the plane of μ integration were the poles, equal to radicals of the equation $1 = R_2F$. This is not so in reality, inasmuch as in the plane μ, there are branching points of the sub-integral expression. The unaccounting of this leads to the consequence that the solution (12) does not include lateral waves, which will propagate in lower media toward both sides of the boundary of nonuniformity.* In this region the effect of quicky-damping lateral waves will be insignificant.

It may be shown that it should not be difficult to obtain a formula similar to (12) by the induction method for the case of a layout consisting of n uniform sections. At formula (12) deduction approximate expression for α were used, which is not compulsory

* see addendum

since the precision of formula (12) may be improved by introducing under the sum expressions of the type $\sqrt{(k_{21}^2/k_0^2)-\nu^2}$ and so for. The transition to approximate formulas for α means that, besides terms $(\alpha_1-\alpha_2)^2$, rejected are terms proportional to α_1^2 and α_2^2. Such approximation is natural, for it always takes place, whenever $R_j \approx 1-(2k_0/\nu k_j)$. Therefore, formula (12) is valid, when in all the essential integration region at the lower boundary the condition (2), in which only the first addend is retained under the radical, is valid for the quantities ψ, ψ_1, ψ_2.

Estimate of Corrections linked with the Variation of the Geomatrical Form of a Piecewise - Uniform Waveguide

Formula (12) for a piecewise-uniform waveguide remains also valid in the case of a spherical waveguide. It is possible that in a uniform spherical waveguide, just as in a plane case, it may be represented in the form of the sum of normal waves, whose eigenvalues have been "corrected" for sphericity. The corrections $\Delta\nu_n$ are computed by means of the formula

$$\frac{\Delta\nu_n}{\nu_n}=-\frac{1}{\nu_n a}\lim_{1/a\to 0}\frac{\partial F/\partial a^{-1}}{\partial F/\partial \nu}, \qquad (13)$$

where a is the radius of waveguide's lower boundary ; $F[C(1/a),\ 1/a]=0$ is the equation of poles, accounting for the sphericity, and also

If a is equal to a_1 and a_2 in the first and second sections, formula (12) is transformed as follows : Instead of r_1 and r_2, the exponent index must contain lengths measured along a distorted surface, and the values ν_n and μ_m must be corrected for the sphericity with the aid of formula (13). There appears ahead of the sum a factor $\sqrt{\theta/\sin\theta}$, where $\theta=(r_1/a_1)+(r_2/a_2)$.

This result is easily obtainable with the aid of equations (1), (2), similarly to what was done for the terrestrial wave about a sphere [5]. The effect of sloping unevennesses of the waveguide may be investigated the same way as for the terrestrial wave. It is then necessary to take into account the fact that a sloping unevenness is irradiated by a field with a wavelength $\lambda = \lambda_0 [1 - \nu_0^2]^{-1/2}$ (naturally in the zone, where only the zero mode remains). The following step in the accounting of the effect of sloping unevenness consists in expanding the scattered field by eigen-function of the waveguide system, and in finding thereby the corrections for the amplitudes of normal waves.

More complex is the matter of accounting the effect of jump-like variation of waveguide's height. In that case, the integration surface S in the equation (1) will contain an additional addend S_2, whose influence may be estimated if the lengths of the uniform sectors are great. It is then necessary to form the first terms of the iteration series. Contrary to formula (12), a complementary addend will appear in the expression for ψ — the integral by the vertical section, which for small Δh, is about equal to :

$$\Delta\psi \sim k_0 \Delta h \psi_1(r_1)\psi_2(r_2)\sqrt{r_1 r_2/(r_1+r_2)} \qquad (14)$$

Comparing formulas (12) and (14) it is not difficult to obtain that

$$\frac{\Delta\psi}{\psi} = \frac{k\Delta h}{8}\,\frac{\nu_0 - \mu_0}{(k_0/k_{21} - k_0/k_{22})}.$$

Taking into account that the variation of wall's properties of the waveguide (on which $\alpha = 0$), consisting in that α becomes a finite magnitude, is equivalent to the variation of waveguide's height [8], let us write the condition of negligibility of the $\Delta\psi$ effect :

$$\frac{\Delta h}{h} \ll |\nu_0|. \qquad (15)$$

In case of great distances from the non-homogeneity boundary this condition is sufficient and valid.

A similar estimate may be obtained by examining the field by the joining method [10] near the jump of waveguide's height, on whose all walls $(\partial\psi/\partial n) = 0$. Let us transport the origin of the coordinates to the point $r = r_H$ and pass from cylindrical to rectangular coordinates. On one side of the non-homogeneity ψ_A may be represented in the form ($H_1 = a$) :

$$\psi_A = A_0 \nu_{0A}^{-1} + \sum_n A_n \nu_{nA}^{-1} \cos n\pi \frac{z}{a}.$$

On the other side of the non-homogencity ($H_2 = b$):

$$\psi_B = B_0 \mu_{0B}^{-1} + \sum_m B_m \mu_{mB}^{-1} \cos m\pi \frac{z}{b}.$$

Conditions of continuity of ψ and of derivatives $\partial\psi/\partial n$ have the form :

$$\begin{array}{ll} \psi_A = \psi_B & 0 < z < a, \\ \psi'_A = \psi'_B & 0 < z < a, \\ \psi'_B = 0 & a < z < b. \end{array}$$

Hence, for amplitudes of the zero modes we obtain the correlation

$$aA_0 = bB_0, \tag{16}$$

analogus to the inequality (15), since it follows from it that

$$\frac{A_0 - B_0}{A_0} = -\frac{a-b}{b} = \frac{\Delta h}{h}.$$

Within the framework of that scheme one may also compute the effect of normal waves of higher numbers. To that effect it is necessary to utilize expansions by eigen-functions, and also the conditions of continuity which will lead to an infinite system of equations for the coefficients A_i :

$$A_l = \sum_{l}^{\infty} A_l g_{ll}. \tag{17}$$

Coefficients g_{il} are determined by the correlations :

$$q_{lq} = \frac{4\varkappa c}{v_{lA}\pi^3}\left[\frac{(-1)^{l+q+1}}{q^2-l^2}\frac{1}{q^2}(W_{qB}-W_{lB})(1-\delta_{lq})+\delta_{lq}\frac{(-1)^q}{q^2}J_q\right], \tag{18}$$

where c — is the wave velocity ; W and J are the series

$$W_{0B}=\sum_{n=1}^{\infty}\frac{\sin^2 n\pi\alpha}{v_{nB}n^2\alpha^2}, \quad W_{mB}=\sum_{n=1}^{\infty}\frac{\sin^2 n\pi\alpha}{v_{nB}\left(\frac{n^2}{m^2}\alpha^2-1\right)},$$

$$J_q=\sum_{n=1}^{\infty}\frac{\alpha n^2\sin^2 n\pi\alpha}{v_{nB}q^2\left(\frac{n^2}{q^2}\alpha^2-1\right)}, \quad g_{0q}=\frac{4\alpha c}{\pi^2}\frac{(-1)^q}{q^2}W_{qB}.$$

In order to take into account the infinity of the distance from the source to the limit of the non-homogeneity, it is necessary to introduce under the signs of the sum of series W and J the relations $H_0^{(1)}(k_0 r_H v_{nB}) / H_1^{(1)}(k_0 r_H v_{nB})$, and also to conduct certain other transformations of formulas (17)and (18). For great values r_H this relation approaches the unity.

Using asymptotics similarly to formula (11), one may obtain the corrections to the formula with an infinitely remote source, proportional to $1/k_0 r_H$. The system (17) may be "cut" if only the first m normal waves may propagate in the waveguide. Their amplitudes are then calculated approximately.

The above-conducted examination shows that the process of wave propagation in a non-uniform waveguide has a great deal in common with the process of terrestrial wave propagation over the spherical non-homogenous Earth.

The ressemblance consists in the additivity of the effect of separate portions, provided they are sufficiently great. The additivity is then characteristic also of a plane waveguide, which does not take place in the case of a piecewise-uniform course of the terrestrial wave.

Physically this circumstance is explained as follows : It is well known that in the case of a plane course of the terrestrial wave, the propagation process has a fundmentally spatial character. In case of a sphere the part of the field, that diffracts in the shade region, seems to be somehow sliding along the boundary of the division. The upper boundary of the waveguide does not allow the propagation process to take the spatial character. The waveguide localises still more intensively the region forming the field at the observation point in the vicinity of the lower boundary of the division : the great extent of uniform sections, indispensable for additivity, is determined in the case of a plane waveguide by its height.

The author is grateful to Ya. L. Al'pert for the discussion of the results of the present work.

**** E N D ****

I Z M I R A N Entered on 5 December 1961

Translated by ANDRE L. BRICHANT
for the
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

8 July 1962

References../..

REFERENCES

1. D. STRETTON. Teoriya elektromagnetizma, Gostekhizdat, M., 1948.
2. A. D. PETROVSKIY, E. L. FEYNBERG. Radiotekhnika i Elektronika, 5, 385, 1960.
3. F. G. BASS. Ibid. 5, 389, 1960.
4. Yu. K. KALININ, A. D. PETROVSKIY. Doklag na Vsesoyuznom simposiyume po diffraktsii. Odessa, 1960.
5. YU. K. KALININ. Tr. IZMIRAN, 17, 50, 1960.
6. L. M. BREKHOVSKIKH. Volny v slistykh sredakh. Gostekhizdat, M, 1956.
7. G. N. VATSON. Teoriya besselevykh funktsiy. IL, M., 1949.
8. S. V. BORODINA, Yu. K. KALININ, G. A. MILHAYLOVA, D. S. FLIGEL'. Trudy "IZMIRAN", 17, 130, 1960.
9. WINNERY, JEMISSON, Proc. IRE, No. 2, 1944.
10. I. R. WAIT. J. Res. Nat. Bur. Standards, No. 1, 1961

ADDENDUM

* Infrapaginal notes.

* From page 4 :

In this way it is possible to find the exact solution of the Sommerfeld problem [1] on a dipole field near the boundary of two half-spaces, and of the more complex problem of a field in a medium consisting of several layers [6].

* From page 8 :

This is immaterial since formula (12) is valid in the second section at distances of many wavelenghts from the the boundary of non-uniformity.
